

# GLUON-PHOTON MIXING in DENSE QCD

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**Abstract:.** At high baryonic density with the formation of a diquark condensate  $\Delta \neq 0$ , the QCD color symmetry is spontaneously broken. Being massive by the Anderson-Higgs mechanism, gluon and photon should mix together within two linear combinations due to the color nonconservation. Consequently a gluon  $\tilde{G}$  could decay into an  $e^-e^+$  pair via its photon component.

With a low invariant mass (about a few ten MeV) and an extremely narrow width peaking above the continuum background, the purely leptonic decay of a strongly-interacting gluon  $\tilde{G} \rightarrow e^- + e^+$  constitutes a very distinctive signature of the color superconductivity phase. By a similar scenario of gluon- $Z$  mixing, another "missing-energy" decay into invisible neutrinos  $\tilde{G} \rightarrow \nu + \bar{\nu}$  could arise, its amplitude is however  $(\Delta/M_Z)^2$  power-suppressed.

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# 1 Introduction

In these last years, our understanding of QCD at extreme conditions, in particular at high baryon density and low temperature, has been considerably advanced and these extensive developments[1] – [22] are amply discussed in some recent reviews[23].

Our starting point is a simple observation that at sufficiently high density characterized by a chemical potential  $\mu \neq 0$ , quark matter is a color superconductor[1]– [24] similar to the standard Bardeen-Cooper-Schrieffer (BCS) superconductivity of metals. Thus like a Cooper electron pair, the pairing of up and down light quarks either by gluon exchange or by instanton-induced would lead to a diquark condensation  $\Delta \neq 0$  which breaks the color symmetry  $SU_c(3)$  down to an unbroken  $SU_c(2)$  color subgroup.

We are in the so called two flavors superconductivity **2SC** phase with two flavors ( $i, j = 1, 2$  stand for up and down quarks) linked by their three colors ( $\alpha, \beta, \gamma = 1, 2, 3$ ) to form a quark bilinear combination  $\Phi_\gamma = \varepsilon^{ij} \varepsilon_{\alpha\beta\gamma} q_i^\alpha q_j^\beta$  which behaves as a Lorentz scalar-isoscalar complex field belonging to a fundamental representation of the  $SU_c(3)$  group as an  $\bar{3}$  color antitriplet. Its nonzero vacuum expectation value designating the diquark condensate takes the form

$$\langle q_i^\alpha q_j^\beta \rangle \varepsilon^{ij} \varepsilon_{\alpha\beta\gamma} = \Delta_\gamma . \quad (1)$$

QCD gauge invariance implies that the composite  $\Phi_\gamma$  field couples to the eight gluons  $G^a$  via the covariant derivative  $(\nabla_\rho)^\beta_\alpha = \partial_\rho \delta^\beta_\alpha - ig_s G^a_\rho (\lambda^a/2)^\beta_\alpha$  in which  $\lambda^a$  denote the Gell-Mann matrices of  $SU_c(3)$ . Since color is spontaneously broken by the order parameter  $\Delta \neq 0$ , five among the eight  $G^a$  gluons become massive via the Anderson-Higgs-Meissner mechanism, the other three gluons belonging to the unbroken  $SU_c(2)$  color subgroup remain massless, as it should be.

Furthermore when electromagnetism is switched on, the covariant derivative  $\nabla_\rho$  has an additional  $-ieA_\rho(\tau^c)$  term,  $\tau^c$  is the charge flavor matrix defined later in (7). Due to the color symmetry breaking, the colorfull gluons  $G^a$  and the colorless photon  $A$  are necessarily mixed together. The mixing arises from the diquark condensate  $\Delta$  inserted in all possible ways as depicted in Figs.I. This case cannot happen to QCD at zero density since  $\Delta$  being proportional to the chemical potential  $\mu$  like  $\Delta \sim \mu \exp(-3\pi^2/\sqrt{2}g_s)$  according to[3],[4] and tends to zero as  $\mu \rightarrow 0$ . The gluon-photon mixing allows us to define a "physical" mass-eigenstate gluon  $\tilde{G}^a$ ,  $a = 4, \dots, 8$  as a linear combination of two gauge-eigenstates with necessarily the photon  $A$  as one inevitable component. Thus

$$\tilde{G}^a = \cos \theta_a G^a - \sin \theta_a A , \quad (2)$$

the mixing angle  $\theta_a$  is function of the condensate  $\Delta$ , the strong QCD coupling constant  $g_s$  and the quark electromagnetic coupling constants  $eQ_j$  ( $Q_u = 2/3$  and  $Q_d = -1/3$ ) as will follow below. The orthogonal combination

$$\tilde{A} = \cos \theta_a A + \sin \theta_a G^a \quad (3)$$

is the "rotated" physical photon in the **2SC** phase. The  $a = 8$  gluon case associated with the diagonalized  $\lambda^8$  matrix has been considered in[5].

The situation is very similar to the standard electroweak interactions in which the massive  $Z$  weak neutral boson is a linear combination of two gauge boson eigenstates, i.e. the hypercharge isosinglet  $B$  is mixed with the third component  $W^3$  of the  $SU_L(2)$  isotriplet  $\mathbf{W}$  to build up the  $Z$ , thus

$$Z = \frac{g}{\sqrt{g^2 + g'^2}} W^3 - \frac{g'}{\sqrt{g^2 + g'^2}} B , \quad (4)$$

the ratio  $g'/\sqrt{g^2 + g'^2}$  is defined as  $\sin \theta_W$ , where  $\theta_W$  is the Weinberg angle. Neither the  $W^3$  nor the  $B$  has a well-defined mass, only their mixture  $Z$  is a mass-eigenstate by the diagonalization procedure.

The gluon-photon mixing ( $\theta_a, \tilde{G}$ ) shares with the standard electroweak interactions ( $\theta_W, Z$ ) and the BCS superconductivity a common crucial point: respectively in these three phenomena, gauge symmetries are spontaneously broken with nonzero vacuum expectation values  $\langle u^\alpha d^\beta - d^\alpha u^\beta \rangle \neq 0$  of the up-down diquark,  $v \neq 0$  of the elementary scalar Higgs field and  $\langle e^- e^- \rangle \neq 0$  of the Cooper electron pair.

We also note that in the standard electroweak interactions, the decays of  $Z$  come from those of both  $W^3$  and  $B$ . For instance the  $Z$  decaying into a neutrino pair is due only to its  $W^3$  component, and the decay of  $Z$  into a right-handed electron pair is due only to its  $B$  part. Similarly, a massive gluon  $\tilde{G}$  could decay electromagnetically into  $e^+ + e^-$  via its photon component as shown by (2), otherwise this process is strictly forbidden in non dense media (chemical potential  $\mu = 0$ ). This lepton pair production by copious gluons in quark matter may provide a remarkable signature of the **2SC** phase transition in neutron star cores and perhaps under certain circumstances in heavy ion collisions.

Our next task is devoted to an estimate of the gluon mass  $\tilde{M}_G$ , its mixing angle and its lepton pair decay width  $\Gamma(\tilde{G} \rightarrow e^+ + e^-)$ . We will see that this lepton pair, with a small invariant mass about 90 MeV and an extremely narrow width is definitely distinct from the continuum Drell–Yan background and the leptonic decays of the well-known vector mesons  $\rho(770), \omega(782)$  with their peaks and tails modified by medium effects[25].

## 2 Gluon, Photon Masses and their Mixing

The traditional way to estimate the photon and gluon masses as well as their mixing in dense matter is to consider their self-energies via their corresponding polarization tensors  $\Pi_{\rho\sigma}$ . The calculational method adopted here follows from[22]. The Lagrangian for gluon and photon interacting with quarks may be put in the form

$$\mathcal{L}_{\text{int}} = g_s G_\rho^a \mathcal{J}_\rho^a + e A_\rho J_\rho^c. \quad (5)$$

The quark currents  $\mathcal{J}_\rho^a$  for QCD and  $J_\rho^c$  for QED, respectively coupled to the gluon  $G_\rho^a$  and the photon  $A_\rho$  with the coupling constants  $g_s$  and  $e$ , are given by

$$\mathcal{J}_\rho^a = \bar{\Psi}^A (\gamma_\rho \lambda^a) \Psi_B, \quad \text{and} \quad J_\rho^c = \bar{\Psi}^A (\gamma_\rho \tau^c) \Psi_B, \quad (6)$$

where  $\tau^c$  is a  $2 \times 2$  charge matrix in the flavor space that we introduce together with another anti-symmetric  $\varepsilon$  matrix,

$$\tau^c = \begin{pmatrix} Q_u & 0 \\ 0 & Q_d \end{pmatrix}, \quad \varepsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \quad (7)$$

The quark-quark pairing term of the effective Lagrangian for the quarks in the condensate may be written as[22]

$$\mathcal{L}_{\text{quark}} = \bar{\Delta}^{AB} \Psi_B \Psi_A + \bar{\Psi}^A \bar{\Psi}^B \Delta_{BA}, \quad (8)$$

the indices  $A, B$  stand collectively for  $i, j$  flavors,  $\alpha, \beta$  colors, and  $\mathcal{A}, \mathcal{B}$  Dirac spinor indices on which operates  $\gamma_5 \mathcal{C}$ ,  $\mathcal{C} = i\gamma^2 \gamma^0$  being the charge conjugation operator. Thus  $A = (i, \alpha, \mathcal{A}), B = (j, \beta, \mathcal{B})$ , and

$$\Delta_{AB} = (\gamma_5 \mathcal{C})_{\mathcal{A}\mathcal{B}} \varepsilon_{ij} \varepsilon_{\alpha\beta} \gamma \Delta^\gamma, \quad \bar{\Delta}^{AB} = \gamma_0 \Delta_{AB}^\dagger \gamma_0. \quad (9)$$

As an example, let us consider in some details the gluon-photon mixing manifested through the correlation function  $\Pi_{\rho\sigma}^{\gamma G^a}$ :

$$\begin{aligned} \Pi_{\rho\sigma}^{\gamma G^a}(x-y) &\equiv \langle 0|T [\mathcal{J}_\rho^a(x) J_\sigma^c(y)] |0\rangle = \langle 0| T \left\{ \overline{\Psi}^A(x) (\gamma_\rho \lambda^a)_A^B \Psi_B(x) \overline{\Psi}^C(y) (\gamma_\sigma \tau^c)_C^D \Psi_D(y) \right. \\ &\quad \times \exp \int dz \left[ \overline{\Delta}^{KL} \Psi_L(z) \Psi_K(z) + \overline{\Psi}^K(z) \overline{\Psi}^L(z) \Delta_{LK} \right] \left. \right\} |0\rangle . \end{aligned} \quad (10)$$

The development of the exponential to second order in  $\Delta$  leads to

$$\begin{aligned} \Pi_{\rho\sigma}^{\gamma G^a}(x-y) &\approx \frac{1}{2} \int dz \int du \langle 0| T \left\{ \overline{\Psi}^A(x) (\gamma_\rho \lambda^a)_A^B \Psi_B(x) \overline{\Psi}^C(y) (\gamma_\sigma \tau^c)_C^D \Psi_D(y) \right. \\ &\quad \times \left[ \overline{\Delta}^{KL} \Psi_L(z) \Psi_K(z) + \overline{\Psi}^K(z) \overline{\Psi}^L(z) \Delta_{LK} \right] \times \left[ \overline{\Delta}^{MN} \Psi_N(u) \Psi_M(u) + \overline{\Psi}^M(u) \overline{\Psi}^N(u) \Delta_{NM} \right] \left. \right\} |0\rangle , \end{aligned} \quad (11)$$

which can be rewritten as

$$\begin{aligned} &(\gamma_\rho \lambda^a)_A^B (\gamma_\sigma \tau^c)_C^D \overline{\Delta}^{KL} \Delta_{NM} \\ &\times \int dz \int du \langle 0| T \left\{ \overline{\Psi}^A(x) \Psi_B(x) \overline{\Psi}^C(y) \Psi_D(y) \Psi_L(z) \Psi_K(z) \overline{\Psi}^M(u) \overline{\Psi}^N(u) \right\} |0\rangle . \end{aligned} \quad (12)$$

When the Wick's theorem for the time-order products is applied to the above equation and the Fourier transformation in momentum space is made, we get three following contributions written in a rather compact form and depicted respectively by the three diagrams of Figs.I. They are

$$\Sigma_1 \times (\lambda^a)_A^B \Delta_{BC} \overline{\Delta}^{CD} (\tau^c)_D^A , \quad \Sigma_2 \times (\lambda^a)_A^B (\tau^c)_B^C \Delta_{CD} \overline{\Delta}^{DA} , \quad \Sigma_3 \times (\lambda^a)_A^B \Delta_{BC} (\tau^c)_D^C \overline{\Delta}^{DA} , \quad (13)$$

the coefficients  $\Sigma_n$  with  $n = 1, 2, 3$  are obtained from the three corresponding quark loop integrations, an overall factor ( $g_s$ ) must be included. After some algebraic manipulations, we get

$$\Pi_{\rho\sigma}^{\gamma G^a}(p) = -e g_s \text{Tr}(\tau^c) \left( \Delta^\alpha (\lambda^a)_\alpha^\beta \overline{\Delta}_\beta \right) \left[ U_{\rho\sigma}^{(1)}(p) + U_{\rho\sigma}^{(2)}(p) + U_{\rho\sigma}^{(3)}(p) \right] , \quad (14)$$

where

$$\begin{aligned} U_{\rho\sigma}^{(1)}(p) &= \frac{i}{(2\pi)^4} \int d^4k \text{Tr} [\gamma_\rho S(p+k) S(-p-k) S(p+k) \gamma_\sigma S(k)] , \\ U_{\rho\sigma}^{(2)}(p) &= \frac{i}{(2\pi)^4} \int d^4k \text{Tr} [\gamma_\rho S(p+k) \gamma_\sigma S(k) S(-k) S(k)] , \\ U_{\rho\sigma}^{(3)}(p) &= \frac{i}{(2\pi)^4} \int d^4k \text{Tr} [\gamma_\rho S(p+k) S(-p-k) \gamma_\sigma S(-k) S(k)] , \end{aligned} \quad (15)$$

and  $S(k)$  in (15) being the quark propagator. Similarly we obtain for the photon-photon and gluon-gluon self-energy tensors the following expressions:

$$\Pi_{\rho\sigma}^{\gamma\gamma}(p) = 2e^2 \left\{ \text{Tr} [(\tau^c)^2] (\Delta^\alpha \overline{\Delta}_\alpha) \left[ U_{\rho\sigma}^{(1)}(p) + U_{\rho\sigma}^{(2)}(p) \right] - \text{Tr} (\tau^c \varepsilon \tau^c \varepsilon) [\Delta^\alpha \overline{\Delta}_\alpha] U_{\rho\sigma}^{(3)}(p) \right\} , \quad (16)$$

and

$$\begin{aligned} \Pi_{\rho\sigma}^{G^a G^b}(p) &= 2g_s^2 \left\{ \text{Tr} [\lambda^a \lambda^b] (\Delta^\alpha \overline{\Delta}_\alpha) \left[ U_{\rho\sigma}^{(1)}(p) + U_{\rho\sigma}^{(2)}(p) - U_{\rho\sigma}^{(3)}(p) \right] \right. \\ &\quad \left. + \Delta^\alpha (\lambda^a \lambda^b)_\alpha^\beta \overline{\Delta}_\beta \left[ U_{\rho\sigma}^{(3)}(p) - U_{\rho\sigma}^{(2)}(p) \right] + \Delta^\alpha (\lambda^b \lambda^a)_\alpha^\beta \overline{\Delta}_\beta \left[ U_{\rho\sigma}^{(3)}(p) - U_{\rho\sigma}^{(1)}(p) \right] \right\} . \end{aligned} \quad (17)$$

It remains to compute the loop integrations

$$U_{\rho\sigma}^{(n)}(p) = \left( g_{\rho\sigma} - \frac{p_\rho p_\sigma}{p^2} \right) U^{(n)}(p).$$

We estimate them at zero temperature,  $\mu = 0$ , and get for  $p = 0$  :

$$U_{\rho\sigma}^{(1)}(0) = U_{\rho\sigma}^{(2)}(0) = U g_{\rho\sigma} \quad , \quad U_{\rho\sigma}^{(3)}(0) = V g_{\rho\sigma} \quad , \quad (18)$$

with

$$U = \frac{1}{2}V = \frac{1}{8\pi^2} \ln \left( \frac{\Lambda^2 + m^2}{m^2} \right) \quad , \quad (19)$$

where  $\Lambda$  is the momentum cut-off and  $m$  being the effective quark mass. The three polarization tensors  $\Pi_{\rho\sigma}^{\gamma G^a}$ ,  $\Pi_{\rho\sigma}^{\gamma\gamma}$  and  $\Pi_{\rho\sigma}^{G^a G^b}$  obtained above in (14), (16) and (17) give the following terms to the Lagrangian of the gauge-eigenstates  $A$  and  $G^a$  :

$$\mathcal{L}_{\text{gauge}} = \frac{1}{2} [m_\gamma^2 (A_\rho)^2 + m_a^2 (G_\rho^a)^2 + f_a A_\rho G_\rho^a] \quad , \quad (20)$$

for any gluon  $a = 4 \dots 8$ . We find

$$\begin{aligned} m_\gamma^2 &= 4e^2 |\Delta^2| \{ 2\text{Tr} [(\tau^c)^2] U - \text{Tr} [\tau^c \varepsilon \tau^c \varepsilon] V \} \quad , \\ m_a^2 &= 4g_s^2 \left\{ |\Delta^2| \text{Tr} [(\lambda^a)^2] (2U - V) + 2\Delta^\alpha [(\lambda^a)^2]_\alpha^\beta \overline{\Delta}_\beta (V - U) \right\} \quad , \\ f_a &= -2eg_s \Delta^\alpha [\lambda^a]_\alpha^\beta \overline{\Delta}_\beta \text{Tr}(\tau^c) (2U + V) \quad . \end{aligned} \quad (21)$$

Since  $\Delta_\gamma$  can be always made real and arranged along the third color axis ( $\Delta_\gamma = \Delta \delta_\gamma^3$ ) without loosing any generality[6], it turns out from (19), (20) and (21) that

$$m_4 = m_5 = m_6 = m_7 = \sqrt{\frac{3}{4}} m_8 \equiv M_G \quad , \quad (22)$$

in agreement with[6], so in the following only a common  $M_G$  is kept.

The  $\mathcal{L}_{\text{gauge}}$  in (20) may be conveniently put in a diagonalized form with the "physical" mass-eigenstates  $\tilde{A}$  and  $\tilde{G}$ ; their respective masses are denoted by  $\tilde{m}_\gamma$  and  $\tilde{M}_G$ :

$$\mathcal{L}_{\text{gauge}} = \frac{1}{2} [\tilde{m}_\gamma^2 (\tilde{A}_\rho)^2 + \tilde{M}_G^2 (\tilde{G}_\rho)^2] \quad . \quad (23)$$

The mass-eigenstates  $\tilde{A}$  and  $\tilde{G}$  are linear combinations of the gauge-eigenstates  $A$  and  $G$  through

$$\begin{aligned} \tilde{A} &= A \cos \theta + G \sin \theta \quad , \\ \tilde{G} &= -A \sin \theta + G \cos \theta \quad , \end{aligned} \quad (24)$$

where the mixing angle  $\theta$  is given by

$$\sin^2 \theta = \frac{1}{2} \left[ 1 - \frac{M_G^2 - m_\gamma^2}{\sqrt{(M_G^2 - m_\gamma^2)^2 + f^2}} \right] \quad . \quad (25)$$

Moreover, since  $M_G^2, m_\gamma^2$  and  $f$  are all proportional to  $\Delta^2$ , by factorization the mixing angle  $\theta$  depends only on the ratio  $e/g_s$  and not on  $\Delta$ . The well-defined masses of the mass-eigenstates  $\tilde{G}$  and  $\tilde{A}$  are

$$\begin{aligned}\tilde{M}_G^2 &= \frac{1}{2} \left[ M_G^2 + m_\gamma^2 + \sqrt{(M_G^2 - m_\gamma^2)^2 + f^2} \right], \\ \tilde{m}_\gamma^2 &= \frac{1}{2} \left[ M_G^2 + m_\gamma^2 - \sqrt{(M_G^2 - m_\gamma^2)^2 + f^2} \right].\end{aligned}\quad (26)$$

Since the parameter  $f$  is definitely nonzero from (21), let us remark that the mixing angle  $\theta$  never vanishes no matter how small  $\tilde{m}_\gamma$  is, including its zero value. It turns out that in some particular case, the mixing parameter  $f$  could happen to be  $2M_G m_\gamma$ , then  $\tilde{m}_\gamma = 0$ , the "rotated" physical photon  $\tilde{\gamma}$  could be massless[5] even in dense matter, although its associated gauge-eigenstate photon  $A$  has  $m_\gamma \neq 0$ . In this case,  $\sin \theta = m_\gamma / \sqrt{M_G^2 + m_\gamma^2} \sim e / \sqrt{g_s^2 + e^2}$ . This situation is just as in the standard electroweak interactions where neither the weak  $T_3$  nor the hypercharge  $Y$  cancels the Higgs vacuum expectation value  $v$  but only does their linear combination  $Q = T_3 + Y/2$ . The symmetry generated by  $Q$  is therefore unbroken, thus making the photon massless and  $\sin \theta_W = g' / \sqrt{g^2 + g'^2}$ . Similarly here[5], a linear combination of the color  $T_8$  and the electric charge  $Q$  is left unbroken, making  $\tilde{m}_\gamma = 0$ . There is an one to one correspondence between the color superconductivity and the standard electroweak interactions:  $(\tilde{G}, \tilde{\gamma}) \iff (Z, \gamma), (G, \gamma) \iff (W^3, B), (g_s, e) \iff (g, g')$ .

Numerical values for the masses  $\tilde{M}_G, \tilde{m}_\gamma$  depend on  $\Delta(\mu)$ , the strong QCD coupling constant  $g_s$ , and the  $U, V$  terms, the latter presumably have a smooth[6] dependence on  $\mu$ . Using  $\Delta \approx 200$  MeV at  $\mu \approx 300$  MeV and  $\alpha_s = g_s^2/4\pi \approx 0.6$ , we get  $\tilde{M}_G \approx 90$  MeV which is consistent with other evaluations[6]–[8]. Our value of 90 MeV is only an order of magnitude estimate, with large uncertainties.

We also note from (21) that  $m_\gamma/M_G \approx e/g_s \ll 1$ , hence (25) and (26) imply that both  $\tilde{m}_\gamma$  and  $\sin \theta$  are small, in particular

$$\sin \theta \approx \frac{e}{2\sqrt{3}g_s} \approx 0.032. \quad (27)$$

As given by (24), the  $e^+ + e^-$  decay mode of  $\tilde{G}$  is due to its photon component which carries a  $(-\sin \theta)$  coefficient. Its decay width  $\Gamma(\tilde{G} \rightarrow e^+ + e^-)$  is computed to be

$$\begin{aligned}\Gamma(\tilde{G} \rightarrow e^+ + e^-) &= \frac{e^2 \sin^2 \theta}{12\pi} \tilde{M}_G \left( 1 + \frac{2m_e^2}{\tilde{M}_G^2} \right) \sqrt{1 - \frac{4m_e^2}{\tilde{M}_G^2}} \\ &\approx \frac{\alpha_{\text{em}}^2}{36 \alpha_s} \tilde{M}_G \approx 0.27 \text{ KeV} \approx 4 \times 10^{17} / \text{sec}.\end{aligned}\quad (28)$$

To estimate the corresponding  $B(\tilde{G} \rightarrow e^+ + e^-)$  branching ratio

$$B(\tilde{G} \rightarrow e^+ + e^-) \equiv \frac{\Gamma(\tilde{G} \rightarrow e^+ + e^-)}{\Gamma(\tilde{G} \rightarrow \text{all})},$$

we assume  $\Gamma(\tilde{G} \rightarrow \text{all}) \approx \cos^2 \theta \Gamma(\tilde{G} \rightarrow q + \bar{q}) \approx \alpha_s \tilde{M}_G / 3$ . Thus  $B(\tilde{G} \rightarrow e^+ + e^-) \approx \alpha_{\text{em}}^2 / 12 \alpha_s^2 \approx 10^{-5}$ .

### 3 Summary and Outlook

As is well-known, statistical treatment of QCD indicates that at high densities, the quarks and gluons will become deconfined, leading to a new state of matter, the so-called Quark-Gluon Plasma (QGP).

This plasma is however ephemeral and soon hadronizes into mesons and baryons so the problem is to find out what are the QGP distinctive signatures. Candidates for these signatures as amply discussed in the literature (see for instance[23],[25]–[30]) are strangeness enhancement,  $J/\psi$  suppression, enhancement in the continuum low mass region  $250 \text{ MeV} < M_{\ell\ell} < 700 \text{ MeV}$  of the lepton pair invariant mass  $M_{\ell\ell}$  spectra,  $\ell^\pm$  stands for both muon and electron. See also[23] for astrophysical signatures.

We suggest here that another signature of the phase transition is provided by an extremely narrow peak at the  $e^+e^-$  invariant mass  $M_{e^+e^-} \approx 90 \text{ MeV}$ .

In summary, at high density with the chemical potential  $\mu \neq 0$ , the QCD color symmetry is spontaneously broken by the formation of a diquark condensate. This is similar to the BCS superconductivity for which no matter how weak is the attractive interaction between electrons due to phonic vibrations, the Cooper electron pair condensate breaks the QED gauge symmetry, thus giving a Meissner mass to the photon. Furthermore, because of the diquark condensate  $\Delta \neq 0$ , the mixing parameter  $f$  in (22) is shown to be definitely nonzero, therefore the gluon and photon must necessarily mix inside a massive gluon  $\tilde{G}$ , exactly as in the standard electroweak interactions where the vacuum expectation value  $v \neq 0$  of the elementary Higgs field necessarily mix the gauge-eigenstates  $W^3$  and  $B$  to build up the mass-eigenstate weak neutral  $Z$  boson. The  $B$  component of the resulting  $Z$  is responsible for its unusual decay into right-handed electron + left-handed positron  $Z \rightarrow e_R^- + e_L^+$ , as well the  $W^3$  component of  $Z$  is responsible of the  $Z \rightarrow \nu + \bar{\nu}$  mode.

Similarly, the photon component of  $\tilde{G}$  implies the existence of the *purely leptonic gluon decay*. The gluon-photon mixing scheme which yields  $\tilde{G} \rightarrow e^+ + e^-$  seems to provide a remarkable signature for the color superconductivity phase transition. Because of the small gluon mass  $\approx 90 \text{ MeV}$ , only the decay  $e^+ + e^-$  is available, the other mode  $\mu^+ + \mu^-$  is absent, in contrast to the Drell–Yan continuum dileptons  $\ell^+ + \ell^-$  spectra emanating from the usual sources such as quark + antiquark annihilation, quark + gluon Compton scattering, gluon+gluon fusion, supplemented by medium effects[25] on known vector mesons  $\rho(770)$ ,  $\omega(782)$ . In the gluon-photon mixing mechanism, the  $e^+ + e^-$  spectrum has only one sharp peak at low mass  $\approx 90 \text{ MeV}$ . Although the  $\tilde{G} \rightarrow e^+ + e^-$  branching ratio is small  $\approx 10^{-5}$ , however since gluons are so abundant in quark matter, this remarkable leptonic gluon decay mode may be hopefully observable.

Finally we remark that with a diquark condensate, when the electroweak interactions are considered together with QCD, the  $Z$  neutral boson, like the photon, should also mix with the gluon, hence the mode  $\tilde{G} \rightarrow \nu + \bar{\nu}$  could occur. Its amplitude is however  $(\Delta/M_Z)^2$  power-suppressed, otherwise the gluonic decay into invisible neutrinos, manifested by a "missing-energy" reaction, would be another surprising signature of the quark matter phase transition.

Also at sufficiently high density for which the strange quark mass may be neglected ( $\mu \gg m_s$ ), we are in the so called color-flavor locking phase **CFL** with three massless flavors locked to their three colors, the basics of purely leptonic decays of strongly-interacting massive gluons remain unchanged.

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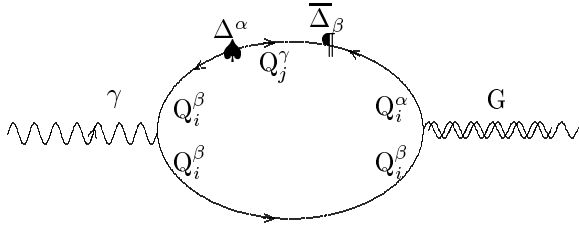


Fig.I.1

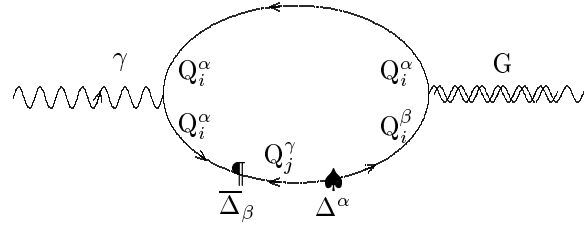


Fig.I.2

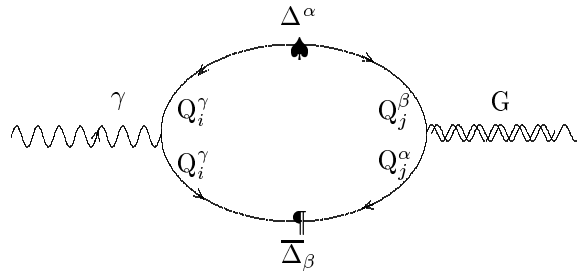


Fig.I.3

### Figure Caption

Figs.I Gluon-Photon mixing